defines the conjugate intensive parameters T (temperature), p (pressure), and ξ as the derivatives of $\psi(s,v,A)$ with respect to (s,v,A), respectively.

If a superscript 0 indicates uniform equilibrium values and a superscript 1 indicates the small perturbations caused by initial and/or boundary conditions, the pertinent linearized equations read

$$D\xi'/Dt = -L^0A' \qquad Ds'/Dt = 0$$
$$Dv'/Dt = v^0\nabla\cdot\mathbf{V}' \qquad (D\mathbf{V}'/Dt) + v^0\nabla p' = 0 \qquad (3)$$

where $D/Dt = (\partial/\partial t) + \mathbf{V}^0\cdot\nabla$, and $\mathbf{V}$ is the velocity vector.

The phenomenological coefficient L^0 is positive, due to the positive character of the entropy production. System (3) is to be implemented by the relations giving the dependent unknowns p' and ξ' as linear combinations of the fundamental set (s', v', A'). These relations are to be obtained from the two state equations $p = p(s,v,A)$ and $\xi = \xi(s,v,A)$.

Accounting for Eq. (3) and for the fact that $s' = 0$, one gets

$$-p' = \psi_{vA}{}^0A' + \psi_{vv}{}^0v' \qquad (4a)$$
$$A' = (1/L^0)\psi_{AA}{}^0(DA'/Dt) + (1/L^0)\psi_{Av}{}^0(Dv'/Dt) \qquad (4b)$$

where the subscripts indicate the partial derivatives of ψ computed at the equilibrium conditions of the basic flow $(A^0 = 0)$.

By definition [see also Eqs. (1) and (2)],

$$\psi_{vv}{}^0 = -[(\partial p/\partial v)]_{A=0} = (1/v^{02})a_e{}^{02} > 0$$
$$-\psi_{AA}{}^0 = [(\partial\xi/\partial A)]_{A=0} = 1/e_{\xi\xi}{}^0 > 0 \qquad (5)$$

where $a_e{}^0$ is the equilibrium speed of sound pertinent to the basic flow, $e_{\xi\xi}{}^0$ is the second derivative of the specific energy $e(\xi,s,v)$ computed at equilibrium, and the inequalities follow from the thermodynamic stability conditions. Equation (5) permits definition of a "chemical-relaxation time" $(1/\tau) = (1/L^0e_{\xi\xi}{}^0)$ as an essentially positive quantity. Equation (4b) thus can be written as

$$A' = -(1/\tau)(DA'/Dt) + (1/\tau)(\psi_{Av}{}^0e_{\xi\xi}{}^0)(Dv'/Dt) \qquad (6)$$

When the first term on the right-hand side of Eq. (6) can be neglected, one obtains

$$A' \cong (\psi_{Av}{}^0/L^0)(Dv'/Dt) \qquad (7)$$

and subsequent substitution into Eq. (4a) yields

$$p' = -[(a_e{}^0)^2/v^{02}]v' - [v^0(\psi_{Av}{}^0)^2/L^0]\nabla\cdot\mathbf{V}' \qquad (8)$$

thus showing that the effects of chemical reaction can be assimilated to those due to an effective coefficient of volume viscosity defined by

$$\eta_v{}^0 = [v^0(\psi_{Av}{}^0)^2/L^0] = (v^0/L^0)[(\partial\xi/\partial v)]_{A=0}{}^2 > 0 \qquad (9)$$

For wave propagation in a medium at rest $(\mathbf{V}^0 = 0)$, the approximate relation (7) is valid when the ratio $\omega/\tau \ll 1$, where ω is the wave frequency. This is seen immediately by taking the Fourier transform (subscript F) of Eq. (6)

$$A_F' = (i\omega/\tau)A_F' - (i\omega/\tau)(\psi_{Av}{}^0e_{\xi\xi}{}^0)v_F'$$

where i is the imaginary unit. For $(\omega/\tau) \ll 1$, this relation reduces to

$$A_F' \cong -(i\omega/\tau)(\psi_{Av}{}^0e_{\xi\xi}{}^0)v_F'$$

which is nothing but the Fourier transform of Eq. (7). This essentially proves that the rate of change of A' can be neglected with respect to A' itself in Eq. (6) when the ratio between the pertinent characteristic time associated with this rate of change and $1/\tau$ is sufficiently small. It follows, then, rather straightforwardly, that Eq. (7) also will be valid for steady flows provided $[\mathbf{V}_r/\tau l_r] \ll 1$ (where $\mathbf{V}_r$ and l_r are suitable reference velocity and length), that is, when the macroscopic characteristic time $l_r/\mathbf{V}_r$ is much larger than the chemical one.

In this case the wave equation, obtained by scalar multiplication of the last of Eqs. (3) by $\mathbf{V}^0$ and substitution of Dp'/Dt from Eq. (8), is, accounting for Eq. (9),

$$[D(\mathbf{V}^0\cdot\mathbf{V}')/Dt] - a_e{}^{02}\nabla\cdot\mathbf{V}' - \eta_v{}^0(D/Dt)(\nabla\cdot\mathbf{V}') = 0$$

or, in terms of nondimensional quantities,

$$(D\mathbf{V}^0\cdot\mathbf{V}'/Dt) - a_e{}^{02}\nabla\cdot\mathbf{V}' = (1/Rev)\mathbf{V}^0\cdot\nabla(\nabla\cdot\mathbf{V}')$$

where $Rev = (\mathbf{V}_rl_r/v^0\eta_v{}^0)$ is a Reynolds number referred to the equivalent volume-kinematic viscosity $(v^0\eta_v{}^0)$ [Eq. (9)]. This form of the wave equation justifies and, at the same time, defines the limits of the use of singular-perturbation and/or boundary-layer-type techniques in the solution of nonequilibrium flows.

The approach could be extended to flows in which more chemical and/or relaxing processes occur. Much as in the problem of acoustical-wave propagation,[1] one could consider all those processes with relaxation times $1/\tau_1$, much smaller than the relevant convective characteristic time t_M, as contributing to an effective volume viscosity and thus treat explicitly as such only the processes with relaxation times of the same order as t_M. Detailed derivation of the wave equation for these cases will be presented in a future note.

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Location of the Normal Shock Wave in the Exhaust Plume of a Jet

DONALD W. EASTMAN* AND LEONARD P. RADTKE†
The Boeing Company, Seattle, Wash.

A METHOD-OF-CHARACTERISTICS program for calculating the exhaust plume flow field of a single axisymmetric jet recently has been completed.[1] The program assumes inviscid flow, with no mixing along the jet boundary.

Figure 1 shows a plot obtained from this program of the boundary and intercepting shock wave shapes for a jet exhausting into still air. As can be seen, the shock wave suddenly becomes normal at a point downstream. This note presents a new method for determining the location of this normal shock.

An approximate method previously suggested by Adamson and Nicholls[2] stated that the axial location of the normal shock was that point at which the static pressure behind the shock was equal to the receiver pressure. However, in actuality the subsonic flow behind the shock may accelerate to supersonic velocities and then pass through a series of weaker shocks.[3, 4] If this occurs, the preceding method can hold only for the last shock in the series.

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* Research Engineer, Aero-Space Division, Flight Technology Department.

† Research Engineer, Aero-Space Division, Applied Mathematics Organization.

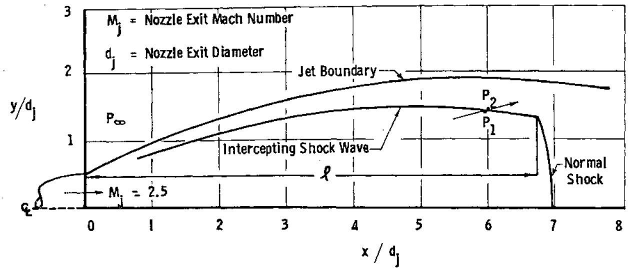

Fig. 1 Exhaust plume for a cold air jet exhausting into still air; $P_j/P_\infty = 12$

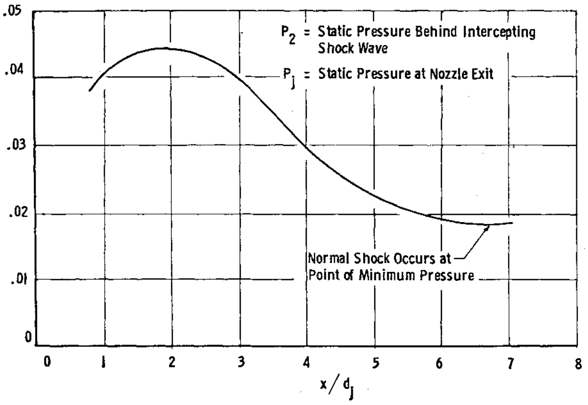

Fig. 2 Variation of the pressure behind the intercepting shock wave for the plume shown in Fig. 1

The characteristics program was used to obtain Fig. 2, which shows a plot of the pressure behind the intercepting shock wave vs x/d_j for the exhaust plume shown in Fig. 1. As can be seen, the pressure first increases and then decreases until an apparent minimum is reached. It is hypothesized that the location of the normal shock wave coincides with the point of minimum pressure. Results from the characteristics program have been compared with existing experimental data, and all results indicate this hypothesis to be correct.

Figure 3 shows a typical comparison of the hypothetical normal shock location with experimental data.[3] Also shown is the predicted normal shock location calculated by applying the method of Adamson and Nicholls[2] to results from the characteristics program. As can be seen, this method predicts that the normal shock will occur at a point further downstream than experimental results indicate. Adamson and Nicholls present a similar curve. However, their curve was obtained using an approximate method to calculate the pressure distribution down the centerline of the exhaust plume. This accounts for the difference between the curve shown in Fig. 3 and the curve shown in Fig. 7c of Ref. 2.

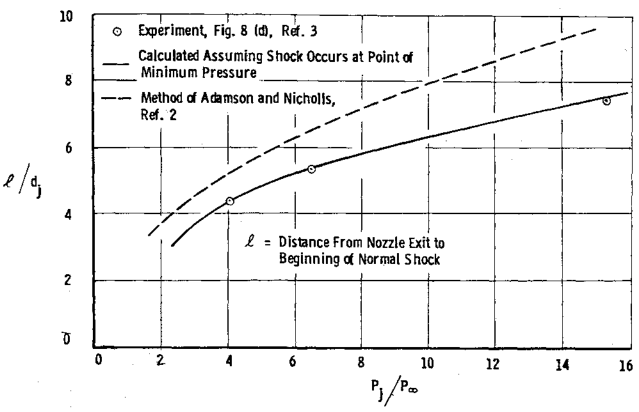

Fig. 3 Comparison of the theoretical and experimental normal shock locations for a cold air jet exhausting into still air; $M_j = 2.5$

Little experimental data for the case of a jet exhausting into a moving external stream are available. However, comparison of available data with results from the characteristics program indicates that the preceding hypothesis also can be used to locate the normal shock for this type flow.

The exact reason for the shock becoming normal, as opposed to striking the axis obliquely, is not known. However, if the normal shock occurs at the point of minimum pressure, there are two important ramifications:

1) The characteristics program assumes inviscid flow, and, therefore, the normal shock is not induced by viscous effects or mixing along the boundary but is determined by the inviscid flow field. Refs. 2, 3, and 5 also reach this same conclusion.

2) The characteristics program is not affected by the flow downstream of the normal shock, and, therefore, the shock location should not be influenced by the flow region behind it. However, Ref. 4 shows that a flame front located behind a normal shock may cause the shock to move upstream.

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Measurements of Thermal Conductivity of Porous Anisotropic Materials

O. E. Tewfik*

Corning Glass Works, Corning, N. Y.

Measurements of the thermal conductivity along several directions and at various average temperature levels of a stainless steel, woven-wire, porous, anisotropic material are described. The material was 0.040-in. thick and was made out of two screens of mesh counts 50 × 250 and 16 × 64 wires/in. by calendering and sintering. Estimated error in the results is ±2%. When compared with the predictions of the thermal ellipse, the results agreed within 1%.

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* Project Engineer. Member AIAA.